

Math 125 Exam 1 Solutions 2/9/10

① T F T F F

②  $\vec{u} \cdot \vec{v} = \langle 2, 1, 3 \rangle \cdot \langle 3, -2, 4 \rangle = 6 - 2 + 12 = 16$

a)  $\vec{u} \times \vec{v} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & -2 & 4 \end{pmatrix} = \hat{i} \begin{vmatrix} 1 & 3 \\ -2 & 4 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} = 10\hat{i} + \hat{j} - 7\hat{k}$

b)  $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta \Rightarrow 16 = \sqrt{14} \sqrt{29} \cos \theta \Rightarrow \theta = \cos^{-1} \frac{16}{\sqrt{14} \sqrt{29}} \approx 37.4^\circ$

③  $\det A = 4 \begin{vmatrix} 4 & 1 \\ 2 & 1 \end{vmatrix} - 3 \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} - 6 \begin{vmatrix} 5 & 2 \\ 4 & 1 \end{vmatrix} = 4 \cdot 2 - 3 \cdot 1 - 6 \cdot (-3) = 23$

④ a)  $\|\vec{r}\| = \|\vec{r} \times \vec{F}\| = \|\vec{r}\| \|\vec{F}\| \sin \theta = \frac{4}{3} \cdot 20 \cdot \sin 60^\circ = \frac{40\sqrt{3}}{3} \approx 23.09 \text{ ft-lb}$

b)  $\vec{r} = \vec{r} \times \vec{F} = \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{4}{3} & 0 & 0 \\ 20 \cos 300 & 20 \sin 300 & 0 \end{pmatrix} = -\frac{40\sqrt{3}}{3} \hat{k} \approx -23.09 \hat{k} \text{ ft-lb}$

⑤  $\left( \begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 4 & 3 & 8 & 16 \\ 2 & 5 & 3 & 14 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 0 & 1 & 6 & 8 \\ 0 & 4 & 2 & 10 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 0 & 1 & 6 & 8 \\ 0 & 0 & -22 & -22 \end{array} \right)$   $x = \frac{1}{2}$   
 $y = 2$   
 $z = 1$

$\left( \begin{array}{ccc|c} 1 & 0 & 2 & 1 & 3 \\ 2 & 1 & 4 & 4 & 10 \\ 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & 0 & 2 & 4 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 4 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 0 & 2 & 1 & 3 \\ 0 & 1 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$   $x = 3 - \alpha - 2\beta$   
 $y = 4 - 2\alpha$   
 $z = \beta$   
 $w = \alpha$

⑥  $\left. \begin{aligned} \vec{F}_1 &= T_1 \cdot \frac{1}{\sqrt{29}} \langle -5, 2 \rangle = T_1 \langle -0.928, 0.371 \rangle \\ \vec{F}_2 &= T_2 \cdot \frac{1}{\sqrt{13}} \langle 3, 2 \rangle = T_2 \langle 0.832, 0.555 \rangle \\ \vec{F}_g &= \langle 0, -20 \rangle \end{aligned} \right\} \begin{pmatrix} -0.928 & 0.832 \\ 0.371 & 0.555 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \end{pmatrix}$

$T_1 = 20.2 \text{ lb}$        $T_2 = 22.5 \text{ lb}$