Math 125 Exam 3 Solutions 4/13/10

(1) $$\frac{6x^2+7x+10}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow 6x^2+7x+10 = A(x^2+1) + (Bx+C)(x+2)$$

$x=-2$: $20 = 5A \Rightarrow A = 4$

$x=0$: $10 = A + 2C \Rightarrow C = 3$

x^2 coeff: $6 = A + B \Rightarrow B = 2$

$$\int \frac{6x^2+7x+10}{(x+2)(x^2+1)}\,dx = \int \frac{4}{x+2}\,dx + \int \frac{2x}{x^2+1}\,dx + \int \frac{3}{x^2+1}\,dx$$

$$= 4\ln|x+2| + \ln(x^2+1) + 3\tan^{-1}x + C$$

(2) $u = x^2 \quad dv = e^x dx$

$du = 2x\,dx \quad v = e^x$

Then

$$\int x^2 e^x dx = x^2e^x - 2\int xe^x dx$$

(IBP again: $u=x \quad dv = e^x dx$, $du = dx \quad v = e^x$)

$$= x^2e^x - 2\left[xe^x - \int e^x dx\right]$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

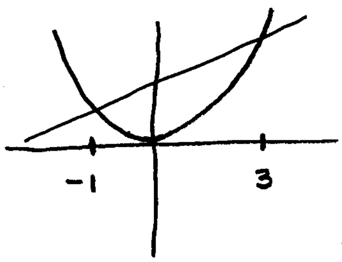
(3) $$\int_2^\infty \frac{dx}{(x+3)^4} = \lim_{R\to\infty} \int_2^R (x+3)^{-4}dx = \lim_{R\to\infty}\left[\frac{-1}{3(x+3)^3}\right]_2^R$$

$$= \lim_{R\to\infty}\left[\frac{-1}{3(R+3)^3} - \frac{-1}{3(2+3)^3}\right]$$

$$= 0 + \frac{1}{3(2+3)^3}$$

$$= \frac{1}{375} \approx 0.00267$$

(4)



Intersection points: $3x^2 = 6x + 9$

$\Rightarrow \quad 3x^2 - 6x - 9 = 0$

$\Rightarrow \quad x^2 - 2x - 3 = 0$

$\Rightarrow \quad (x-3)(x+1) = 0$

$x = 3 \quad x = -1$

a) $M = \int_{-1}^{3} \left[(6x+9) - 3x^2\right] dx = \left[3x^2 + 9x - x^3\right]_{-1}^{3}$

$= 27 + 27 - 27 - (3 - 9 + 1)$

$= 32$

b) $M_y = \int_{-1}^{3} x\left(6x + 9 - 3x^2\right) dx$

c) $M_x = \frac{1}{2} \int_{-1}^{3} \left[(6x+9)^2 - (3x^2)^2\right] dx$

d) $x_{cm} = \dfrac{M_y}{M} \qquad y_{cm} = \dfrac{M_x}{M}$

(5) a) $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi} \int_{0}^{\pi} 1\,dx = \frac{1}{\pi} x \Big|_0^{\pi} = 1$

b) $a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx\,dx = \frac{1}{\pi} \int_0^{\pi} \cos kx\,dx = \frac{1}{k\pi} \sin kx \Big|_0^{\pi} = 0$

(<u>NOT</u> an odd function!)

c) $b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx\,dx = \frac{1}{\pi} \int_0^{\pi} \sin kx\,dx = \frac{-1}{k\pi} \cos kx \Big|_0^{\pi}$

$= \dfrac{-\cos k\pi}{k\pi} - \dfrac{-1}{k\pi} = \dfrac{1 - \cos k\pi}{k\pi}$

d) $b_k = \begin{cases} 0 & \text{if } k \text{ is even} \\ \dfrac{2}{k\pi} & \text{if } k \text{ is odd} \end{cases}$

(Problem 5 was graded on scale of 0-25. Should have been 0-20. I treated it as 20 points + up to 5 pts extra credit.)